Divergence

Suppose $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$
is defined at each (x,y,z) and
differentiable at each (x,y,z)

then the divergence of $\vec{A}$ is denoted by
$\vec{\nabla}.\vec{A}$ and is defined as

$$\vec{\nabla}.\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot\left(A_1\hat{i} + A_2\hat{j} + A_3\hat{k}\right)$$

$$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

It is also denoted as div $\vec{A}$.
So it is a ~~vec~~ ~~sca~~ scalar.

Properties: If $\vec{A}, \vec{B}$ be two differentiable
i) vector functions such that
$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$, $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$
and ϕ is a differentiable scalar field.

1) $\vec{\nabla}\cdot(\vec{A}+\vec{B}) = \vec{\nabla}\cdot\vec{A} + \nabla\cdot\vec{B}$

2) $\vec{\nabla}\cdot(\phi\vec{A}) = (\vec{\nabla}\phi)\cdot\vec{A} + \phi\vec{\nabla}\cdot\vec{A}$

1) H.W.

~~2) $\nabla\cdot(\phi$~~

2). $\vec{\nabla}\cdot[\phi(A_1\hat{i}+A_2\hat{j}+A_3\hat{k})]$

$= \vec{\nabla}\cdot[(\phi A_1)\hat{i}+(\phi A_2)\hat{j}+(\phi A_3)\hat{k}]$

$= \frac{\partial}{\partial x}(\phi A_1) + \frac{\partial}{\partial y}(\phi A_2) + \frac{\partial}{\partial z}(\phi A_3)$

$= A_1\frac{\partial\phi}{\partial x} + \phi\frac{\partial A_1}{\partial x} + \frac{\partial\phi}{\partial y}A_2 + \phi\frac{\partial A_2}{\partial y}$
$+ \frac{\partial\phi}{\partial z}A_3 + \phi\frac{\partial A_3}{\partial z}$.

$= \left(A_1\frac{\partial\phi}{\partial x} + A_2\frac{\partial\phi}{\partial y} + A_3\frac{\partial\phi}{\partial z}\right)$
$+ \phi\left(\frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}\right)$

~~$=(A_1\hat{i}+A_2\hat{j}+A_3\hat{k}))$~~

$= (A_1\hat{i}+A_2\hat{j}+A_3\hat{k})\left(\frac{\partial\phi}{\partial x}\hat{i}+\frac{\partial\phi}{\partial y}\hat{j}+\frac{\partial\phi}{\partial z}\hat{k}\right)$
$+\phi\left(\frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial y}+\frac{\partial A_3}{\partial z}\right)$

~~$= \vec{A}\cdot\nabla\phi + \phi\cdot\nabla\cdot$~~

$= \vec{A}\cdot\vec{\nabla}\phi + \phi\vec{\nabla}\cdot\vec{A}$

<u>Def:</u> If $\vec{A}$ is vector such that $\vec{\nabla}\cdot\vec{A} = 0$, then $\vec{A}$ is called solenoidal.

H.W. Find the constant α such that

$$\vec{A} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+\alpha z)\hat{k}$$

is solenoidal.

H.W.

If $\vec{A} = x^2y\hat{i} + zxy\hat{j} + zx\hat{k}$.

then find $\vec{\nabla}\cdot\vec{A}$ at $(1,-1,5)$